Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities rational or radical expressions or logarithmic or exponential functions).

Graphing Inequalities

When graphing **inequalities**, you use much the same processes you used when graphing equations. The difference is that *inequalities* give you infinitely larger sets of solutions. In addition, your results with inequalities are always expressed using the following terms in relation to another expression:

- greater than ($>$)
- greater than or equal to ($\geq$)
- less than ($<$)
- less than or equal to ($\leq$)
- not equal to ($\neq$).

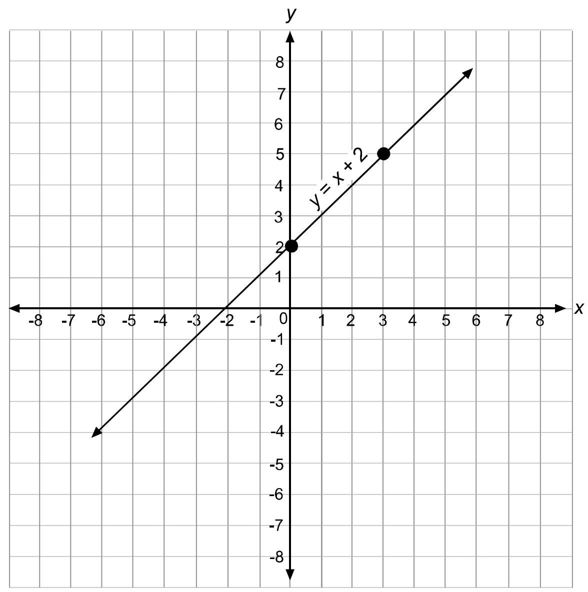
Therefore, we cannot graph an inequality as a line or a point. We must illustrate the entire set of answers by *shading* our graphs.

For instance, when we graph $y = x + 2$ using points, we found by using the table of values below, we get the line seen in Graph 1 below.

Table of Values

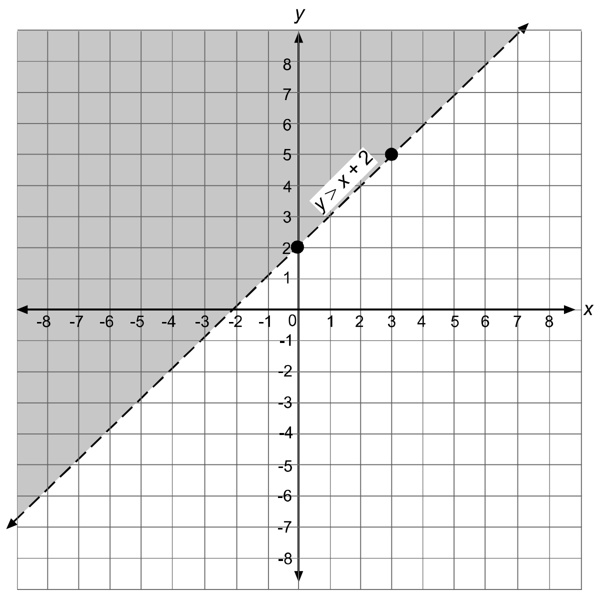
$y = x + 2$	
x	y
0	2
3	5

Graph 1 of $y = x + 2$



But when we graph $y > x + 2$, we use the line we found in Graph 1 as a *boundary*. Since $y \neq x + 2$, we show that by making the boundary line *dotted* (◄- - -►). Then we shade the appropriate part of the grid. Because this is a "greater than" (>) problem, we shade *above* the dotted boundary line. See Graph 2 below.

Graph 2 of $y > x + 2$

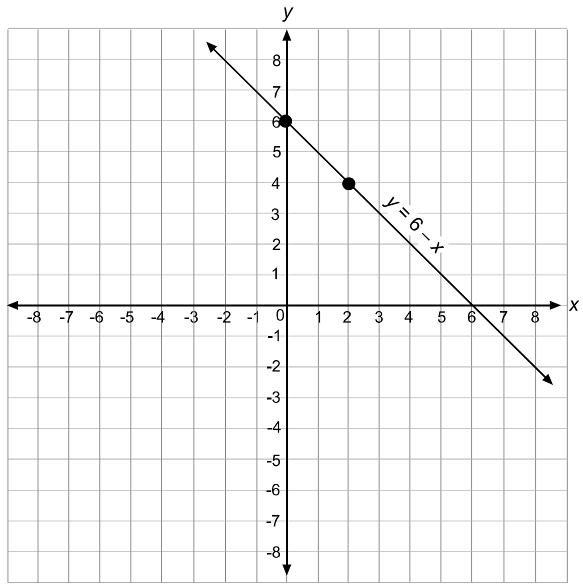


Suppose we wanted to graph $x + y \le 6$. We first transform the inequality so that y is alone on the left side: $y \le 6 - x$. We find a pair of points using a table of values, then graph the boundary line. Use the equation $y = 6 - x$ to find two pairs of points in the table of values. Graph the line that goes through points (0, 6) and (2, 4) from the table of values.

Table of Values

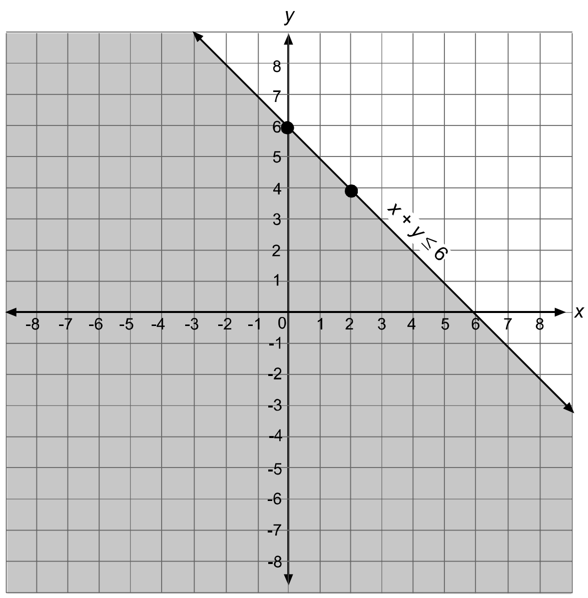
$y = 6 - x$	
x	y
0	6
2	4

Graph 3 of $y = 6 - x$



Now look at the inequality again. The symbol was ≤, so we leave the line solid and shade below the line.

Graph 4 of $x + y \le 6$



Remember: *Change the inequality sign* whenever you *multiply* or *divide* the inequality by a **negative number**.

$< \rightarrow >$ $\quad$ $\leq \rightarrow \geq$

$> \rightarrow <$ $\quad$ $\geq \rightarrow \leq$

Inequality signs always change when multiplying or dividing by *negative numbers*.

Note:

- **Greater than** (>) means to shade **above** or to the **right** of the line.
- **Less than** (<) means to shade **below** or to the **left** of the line.

Test for Accuracy Before You Shade

You can test your graph for accuracy before you shade by *choosing a point* that *satisfies the inequality*. *Choose a point* that falls in the area you are about to shade. Do *not* choose a point *on the boundary line*.

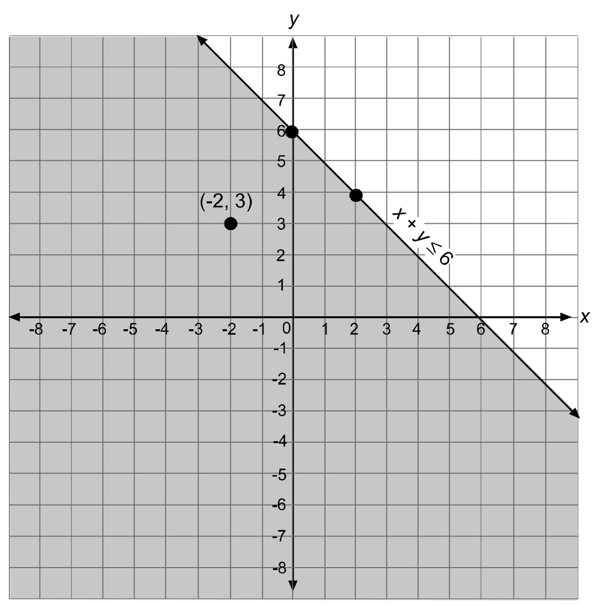
For example, suppose you chose (-2, 3).

$$-2 + 3 \leq 6$$
$$1 \leq 6$$

1. The ordered pair (-2, 3) *satisfies* the inequality.
2. The ordered pair (-2, 3) falls in the area about to be shaded.

Thus, the shaded area for the graph $x + y \leq 6$ is correct.

Graph 4 of $x + y \leq 6$ with Test Point



Graphing Multiple Inequalities

We can graph two or more inequalities on the same grid to find which solutions the two inequalities have in common or to find those solutions that work in one inequality or the other. The key words are "and" and "or." Let's see how these small, ordinary words affect our graphing.

Example 1

Graphically show the solutions for $2x + 3y > 6$ **and** $y \le 2x$.

Note: See how the inequality $2x + 3y > 6$ is transformed in the table of values into the equivalent inequality $y > 2 - \frac{2}{3}x$. Refer to pages 778-781 as needed.

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

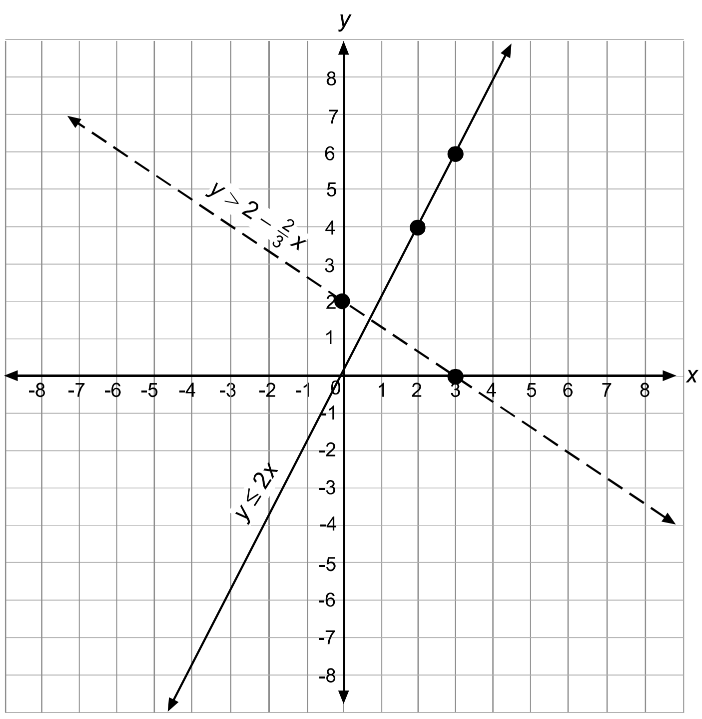
Table of Values

$y > 2 - \frac{2}{3}x$	
x	***y***
0	2
3	0

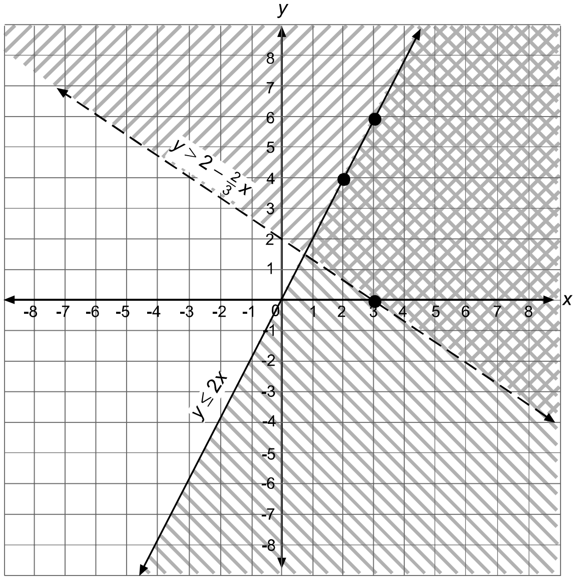
Table of Values

$y \le 2x$	
x	***y***
3	6
2	4

Graph Shows Boundary Lines of $2x + 3y > 6$ and $y \le 2x$



Graph of $2x + 3y > 6$ and $y \le 2x$ with Both Inequalities Shaded



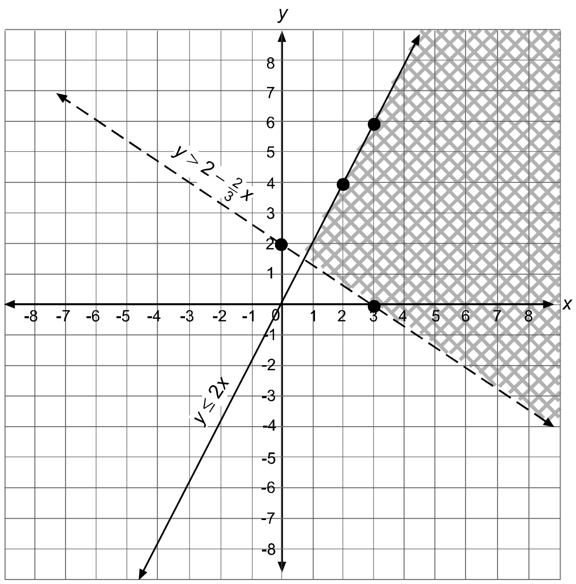
Step 2. Since the 1st inequality is *greater than*, shade *above* the dotted line.

Step 3. Shade the 2nd inequality *below* the solid line using a different type shading or different color.

Step 4. Because this is an "and" problem, we want to have as our solution *only* the parts where both shadings appear at the same time (in other words, where the shadings *overlap*, just as in the **Venn diagrams** in a previous unit). We want to show *only* those solutions that are *valid in both* inequalities *at the same time*.

Step 5. The solution for $2x + 3y > 6$ **and** $y \le 2x$ is shown to the right.

Final Solution for Graph of $2x + 3y > 6$ and $y \le 2x$



Look at the finished graph above.
The point (-1, 1) is *not* in the shaded region. Therefore, the point (-1, 1) is *not* a solution of the intersection of $2x + 3y > 6$ and $y \le 2x$.

Example 2

Let's see how the graph of the solution would look if the problem had been $2x + 3y > 6$ **or** $y \le 2x$.

We follow the same steps from 1 and 2 of the previous example.

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

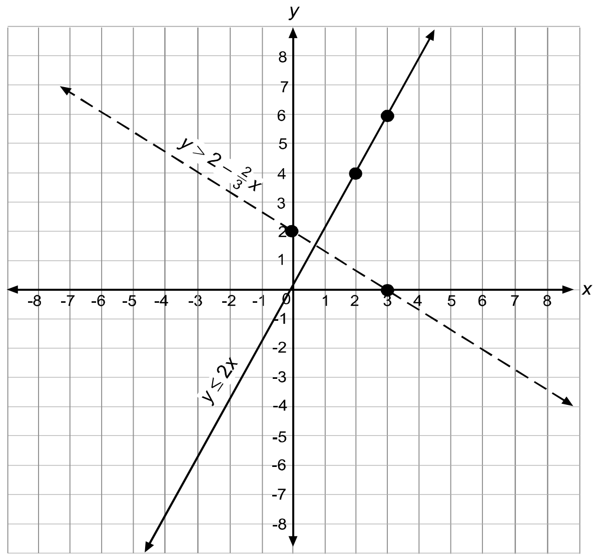
Table of Values

$y > 2 - \frac{2}{3}x$	
x	***y***
0	2
3	0

Table of Values

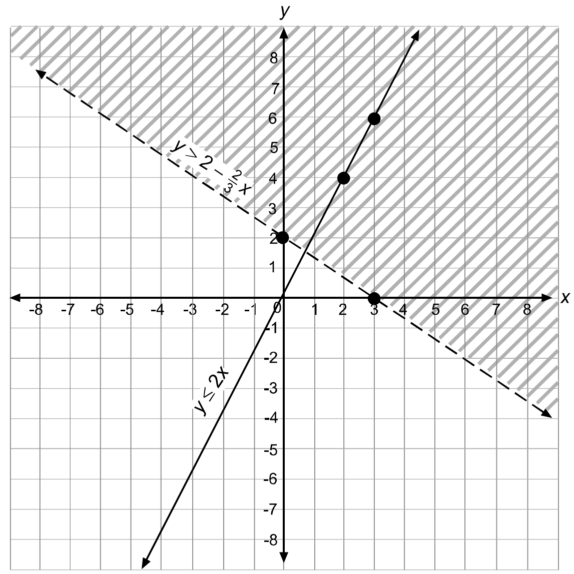
$y \le 2x$	
x	***y***
3	6
2	4

Graph Shows Boundary Lines of $2x + 3y > 6$ or $y \le 2x$



Step 2. Since the 1st inequality is *greater than*, shade *above* the dotted line.

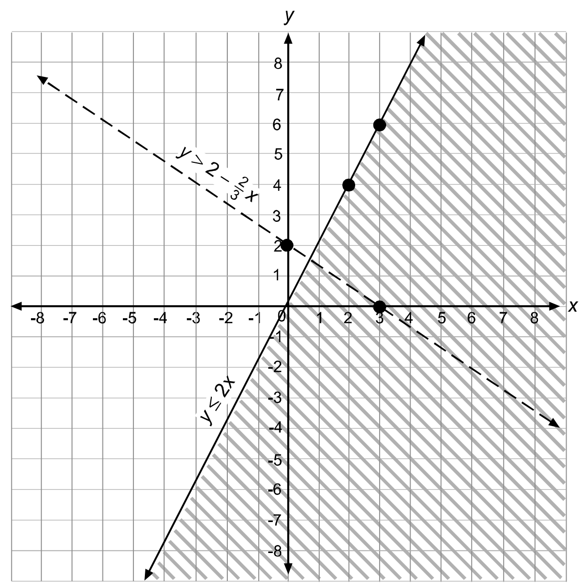
Graph of 2*x* + 3*y* > 6 or *y* ≤ 2*x* with 1st Inequality Shaded



Now we change the process to fit the "*or*."

Step 3. Shade the 2nd inequality *below* the solid line using the same shading as in step 2.

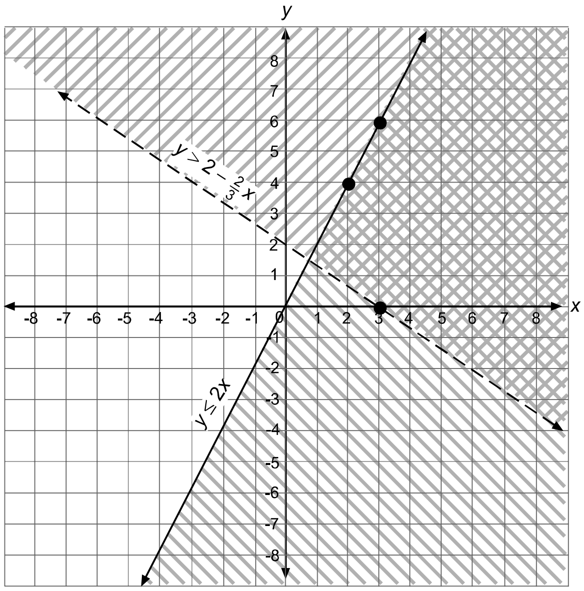
Graph of 2*x* + 3*y* > 6 or *y* ≤ 2*x* with *y* ≤ 2*x* Shaded



Step 4. Because this is now an "or" problem, we want to have as our solution *all* the parts that are shaded. This shows that a solution to *either* inequality is *acceptable*.

Step 5. The solution for $2x + 3y > 6$ **or** $y \leq 2x$ is shown below.

Final Solution for Graph of $2x + 3y > 6$ or $y \leq 2x$



Now it's your turn to practice.